Amplify Desmos Math

Grades K–12

Foundational research

Amplify's commitment

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. To create products that are effective and high-quality, we always start with the research. From this strong foundation, we build programs that provide teachers with powerful tools to help them understand and respond to the needs of every student.

Table of contents

Expert advisors and educator partners 4
Program advisors and contributors 4
Partner organizations 5
Classroom advisors 5

Themes in the research behind Amplify Desmos Math 6

A structured approach to problem-based learning 9
What is problem-based learning, and why is it important? 9
A structured approach to problem-based learning best ensures all aspects of rigor. 11
Conceptual understanding 12
Procedural fluency 13
Modeling and real-world applications 14

Math that motivates 15
Engagement and motivation 15
Fostering math identity 16

Student thinking is valuable and can be made evident 19
Responsive Feedback™ 20
Productive struggle 21
Classroom discourse 22

Access to grade-level understanding for every student, every day 25
Universal Design for Learning 26
Multilingual/English learners 28
Multi-Tiered System of Supports 30
Differentiation 32

References 34

Expert advisors and educator partners

Amplify Desmos Math combines two groundbreaking research and development efforts in K–12 mathematics instruction. The acquisition of Desmos Classroom by Amplify Education in 2022 brought together two distinct, yet complementary, curriculum efforts based on Illustrative Mathematics'® IM K–12 Math™.

A dedicated team with more than 1,900 years of combined classroom teaching and school leadership experience created Amplify Desmos Math with teachers and students in mind. This industry-leading team includes curriculum developers, interaction developers, copy editors, graph specialists, digital innovation specialists, standards and customization specialists, production editors, and many more.

Everyone on the team understands the needs of K–12 teachers and learners and is dedicated to rigorous and accessible mathematics instruction.

Program advisors and contributors

Jason Zimba, Ph.D. A leader in mathematics education, Jason was a founding partner of Student Achievement Partners, an author of the "Publishers' Criteria for Mathematics," and a catalyst in countless initiatives to improve math education nationwide. He is now Chief Academic Officer at Amplify.

Dan Meyer, Ph.D. A longtime advocate for better math instruction and a recipient of the Taylor/Gilbert National Leadership award from the National Council of Supervisors of Mathematics. Dan served as Chief Academic Officer at Desmos, making digital math tools more accessible and engaging for students. He continues to shape the future of math technology as Vice President of User Growth at Amplify.

Fawn Nguyen With over 30 years of classroom experience, Fawn is renowned for her teaching methods in the area of problem-solving. She has shared her knowledge at countless conferences and workshops nationwide. She now leverages her expertise as Director of STEM Initiatives at Amplify.

John W. Staley, Ph.D. A longtime educator and past president of NCSM, John has worked to improve school systems and prepare students for the future. He has also served as chair of the U.S. National Commission on Mathematics Instruction and as a board member for Student Achievement Partners.

Kristin Gray A recipient of the Presidential Award for Excellence in Mathematics and Science Teaching, Kristin is passionate about the value of curiosity in the classroom. Beyond her role as a teacher and coach, she has influenced math education at scale as former Director of K–5 Curriculum at Illustrative Mathematics and current Executive Director of Math at Amplify.

Patrick Callahan, Ph.D. A research mathematician passionate about using assessment to understand student thinking, Patrick co-founded Math ANEX, now a part of Amplify. He has served as statewide co-director of the California Mathematics Project and Senior Research Scientist at WestEd. He is Vice President of Math Education and Assessment at Amplify.

Phil Daro A recipient of the Walter Denham Award from the California Mathematics Council and the Ross Taylor/Glenn Gilbert National Leadership Award from the National Council of Supervisors of Mathematics, Phil is dedicated to rigorous and equitable mathematics instruction. He is a long-time member of the National Assessment of Educational Progress Validity Studies panel.

Partner organizations

Our close collaboration with Desmos Studio–which sets the standard for calculators and digital tools for exploring mathematics–enables us to build delightful and accessible activities in Amplify Classroom, Amplify's teaching and learning platform. **desmos.com**

Math is a language that needs to be developed. Our work with English Learners Success Forum (ELSF) supports the development of all students' language skills with thoughtful integration of strategies and best practices for multilingual and English learners. **elsuccessforum.org**

Note: ELSF does not rate or endorse materials. ELSF encourages all selection of materials to go through a robust adoption process using EL-inclusive criteria.

Multiplication By Heart and other fluency decks by Math for Love are included in Amplify Desmos Math, enabling students to build and maintain essential fluencies in a positive and motivating way. **mathforlove.com**

Classroom advisors

Chris Shore, M.Ed., Secondary Curriculum and Instruction Coordinator, Murrieta Valley Unified School District
Professional learning, intervention, usability, and secondary mathematics

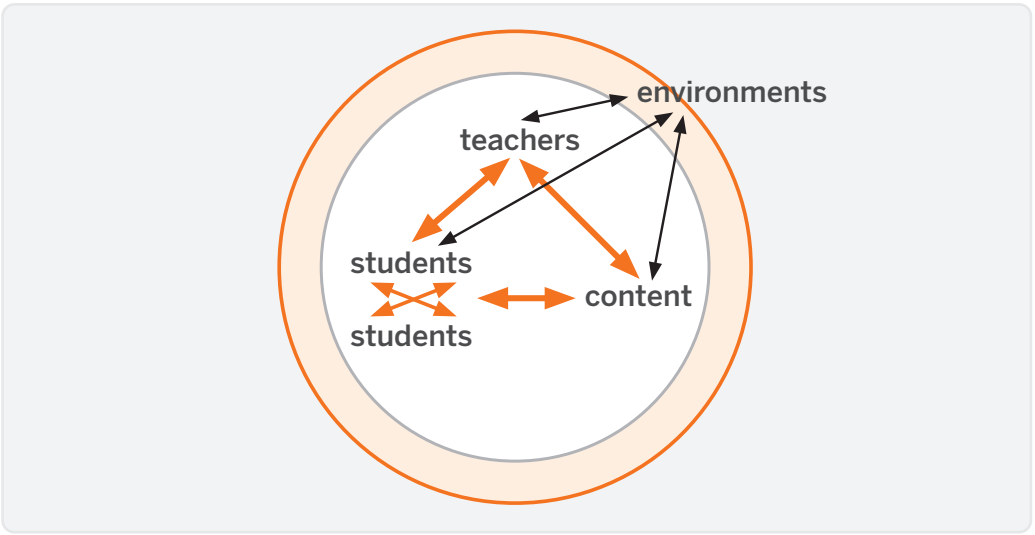
JoAnna Chocooj, Vallejo City Unified School District (retired)
Early childhood mathematics

Leila Sales, Children's Book Author, Publisher, and Consultant
Elementary Unit Stories

Michelle Douglas Meyer, M.A., District Math Facilitator
Oak Creek-Franklin School District, Wisconsin

Themes in the research behind Amplify Desmos Math

Education is one of the most important projects in society, and also one of the most complex. This complexity is reflected in the variety of approaches and findings in contemporary mathematics education research. A useful way to think about both the project of mathematics education as well as its research base is a diagram known as the instructional triangle. The following is just one version of it (Cohen, Raudenbush, & Ball, 2003):



This picture illustrates how, in education, students interact with other students, students interact with teachers, both of them interact with the content, and all three interact with the larger environment, ranging from the school and the district outward to the family and society.

At the heart of the instructional triangle, teachers and students engage with the content in daily lessons. The lessons in Amplify Desmos Math take a structured approach to problem-based learning, providing teachers with clear step-by-step moves to build systematically from students' prior knowledge to grade-level learning. The program thoughtfully combines conceptual understanding, procedural fluency, and application, motivating students with interesting problems they are eager to solve while supporting teachers in guiding students to grade-level outcomes.

Students' interactions with other students are an essential part of the instructional triangle, and Amplify Desmos Math enables productive student collaboration through thoughtfully designed lesson activities and easy-to-use features of the digital platform.

Amplify Desmos Math builds math proficiency for life by helping teachers create an engaged, collaborative math community with students at its center. It's a core program for grades K–12 that combines the best of problem-based lessons, intervention, personalized practice, and assessment into a coherent and engaging experience for both students and teachers.

This document outlines the foundational research upon which the Amplify Desmos Math program was developed, and highlights the academic work behind our conceptualization of high-quality mathematics instruction.

This body of research informed four important themes, which ground Amplify Desmos Math:

1. Math should motivate students.
2. A structured approach to problem-based learning best tends to all aspects of rigor.
3. Student thinking is valuable and can be made evident.
4. Every student should have access to grade-level math, every day.

Amplify Desmos Math builds math proficiency for life by helping teachers create an engaged, collaborative math community with students at its center.

In this resource, you'll find the pedagogical foundation of Amplify Desmos Math, including the following topics:

- Problem-based learning
- Sensemaking and conceptual understanding
- Productive struggle
- Activating prior knowledge and real-world application
- Educator support and training
- Fluency
- Accessibility
- Asset-based learning
- Universal Design for Learning
- Classroom discourse
- Multilingual/English learners
- Content progression coherence
- Core instruction and having a Multi-Tiered System of Supports
- Assessment practices
- The critical role of math-based technology
- Responsive Feedback
- Metacognition
- Fostering math identity
- Motivation
- Student engagement

Mathematics learning is cumulative and occurs when educators continually build on and deepen student knowledge. In order to foster deep and meaningful learning, it's important for teachers to have access to a high-quality mathematics curriculum, including a coherent sequencing of core mathematical ideas across grades and courses (National Council of Teachers of Mathematics, 2014). A rigorous mathematics program empowers teachers and students as they make deep connections across multiple domains of mathematical study and the real world.

The Amplify Desmos Math curriculum is based on the same scope and sequence as Illustrative Mathematics IM K–12 Math, which is highly rated for its focus, coherence, and rigor. (Specific EdReports ratings and details can be found at **edreports.org**.) Materials in Amplify Desmos Math assess and give all students extensive work with grade-level content to fully meet grade-level standards. Assessments are aligned to these standards, including all quizzes and end assessments. Lessons give students meaningful experiences with grade-level content and include practice problems that serve to reinforce and extend understanding of grade-level concepts.

Materials address the major clusters of the grade, have supporting content connected to big ideas, make connections between clusters and domains, and connect content from prior and future grades to grade-level work.

Students who develop proficiency in mathematical problem solving early are better prepared for advanced mathematics and other complex tasks (Lubis et al., 2018). Meaningful discourse and productive struggle are features of learning situations where students understand what's being asked of them and are guided, when necessary, to use the tools in their tool belts to reason effectively.

A structured approach to problem-based learning

What is problem-based learning, and why is it important?

Problem-based learning is a teacher-facilitated instructional approach that situates active learning in a meaningful problem (Hmelo-Silver, 2004; Walker & Leary, 2009). Students draw on background knowledge as they solve real-world problems that have multiple solution pathways (Wirkala & Kuhn, 2011; Hmelo-Silver & Barrows, 2006). These problems have the power to generate rich discussions that promote reflection, application of learning, and a clear understanding of content (Wirkala & Kuhn, 2011).

This instructional approach embodies the eight effective teaching practices outlined by the National Council of Teachers of Mathematics (NCTM). (See page 10.) Problem-based learning fosters a learning environment where mathematical discourse, problem solving, and reasoning are central to the learning environment. Problem-based learning lessons incorporate real-world tasks with clear learning goals. These tasks help students connect various mathematical representations and strategies. As teachers facilitate learning, they pose questions that help students connect prior knowledge to new ideas, communicate their thinking, and deepen their understanding of key mathematics concepts (Smith & Stein, 2011). As students engage in productive struggle, they develop persistence when solving complex problems, an essential part of the learning process.

The highest quality problem-based lessons embody all eight of the National Council of Teachers of Mathematics (NCTM) Teaching Practices:

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

Students benefit from discussing their ideas among peers with different perspectives (Driscoll, 1994; Hausfather, 1996). According to Frank K. Lester, Jr. (2003), instruction is more effective when students grapple actively with the math problems, working in classroom groups as well as individually. There is strong evidence for the efficacy of supporting students as they solve complex problems by monitoring them and reflecting on the process with them (Lubis, et al., 2018). With high-quality problem-based learning, students will:

- Activate background knowledge independently.
- Build a mental library of reasoning strategies.
- Communicate effectively with their teachers and peers.
- Think creatively about situations.
- Persevere in solving problems.
- See themselves as mathematical thinkers and doers.

That's why Amplify Desmos Math units are also designed around a Proficiency Progression™—a model that systematically builds on students' curiosity to develop lasting grade-level understandings. This approach also provides guidance for teachers new to problem-based instruction.

Proficiency Progression
Lessons are designed around what we call the Proficiency Progression, a model that systematically builds on students' curiosity to develop lasting grade-level understanding.

1. Activate students' prior knowledge and curiosity
2. Generate new ideas through collaboration
3. Refine ideas using facilitation tools
4. Guide to grade-level understanding
5. Practice, reinforce, remediate, and extend for lasting understanding

A structured approach to problem-based learning best ensures all aspects of rigor.

By providing teachers with a structured approach to problem-based learning, Amplify Desmos Math empowers all students to build lifelong mathematical proficiency. (See page 12). Each lesson is designed to tell a story, posing problems that invite a variety of approaches, then progressing toward convergence of thinking and a final synthesis of students' understanding of the learning goals. With a structured approach to problem-based learning, students:

- Explore problems that are interesting and raise questions about math.
- Work on problems independently and collaboratively.
- Spend time practicing the math that is most important.
- Receive teacher guidance at key instructional moments.

Amplify Desmos Math supports teachers implementing a problem-based approach by guiding them through observation and facilitation of student discussion. (See page 26.) Lessons proceed through three phases, beginning with the use of a high-level instructional task (Smith & Stein, 2018). The Launch, Monitor, Connect framework discussed in *5 Practices for Orchestrating Mathematics Discussions* (Smith & Stein, 2011) enables students to focus on the math, and provides more time for discussions that foster collaboration and a shared understanding of key mathematics concepts. (The figure below provides an overview of each phase.)

1. Launch
Teacher introduces a high-level task and ensures students have access to materials and understand the key mathematics concepts and language.

2. Monitor
Students work on the task, often in small groups or pairs, using strategies that make sense to them. Teachers visit groups and pose and answer questions to support task exploration.

3. Connect
The class comes together to discuss and summarize student-generated approaches to solving the task. Teachers facilitate a conversation that helps students make connections across solutions and key mathematics concepts.

This Launch, Monitor, Connect framework (along with Instructional Routines and Math Language Routines [see page 28]) provides an approachable entry point for teachers seeking to implement problem-based learning.

Conceptual understanding

Exposure to mathematical topics is not enough for students—they also need to think about and use the mathematics they are learning in different ways. This is the skill of sensemaking. Sensemaking and reasoning are the foundation for mathematical proficiency (Battista, 2017). As students make sense of mathematical ideas, they can apply them in problem-solving and unfamiliar situations. In doing so, students deepen their conceptual understanding of important mathematics concepts. Conceptual understanding—the comprehensive understanding of concepts, operations, representations, and relations (National Research Council, 2001)—is important because it helps students understand the why behind the how, encourages them to explain their mathematical thinking clearly and accurately, enables them to apply what they have learned to the real world, and supports procedural fluency.

In Amplify Desmos Math, students are introduced to engaging problems that help them leverage their current understandings and problem-solving strategies to develop solutions. The program provides an active learning experience that leads students to explore, notice, question, solve, justify, explain, represent, and analyze. Teachers guide this process, supporting synthesis and sensemaking at the end of each lesson.

The National Research Council report "Adding It Up" defined five interwoven strands of mathematical proficiency (2001). They are:

- **Conceptual understanding:** Comprehension of mathematical concepts, operations, and relations.
- **Procedural fluency:** Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.
- **Strategic competence:** Ability to formulate, represent, and solve mathematical problems.
- **Adaptive reasoning:** Capacity for logical thought, reflection, explanation, and justification.
- **Productive disposition:** Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Procedural fluency

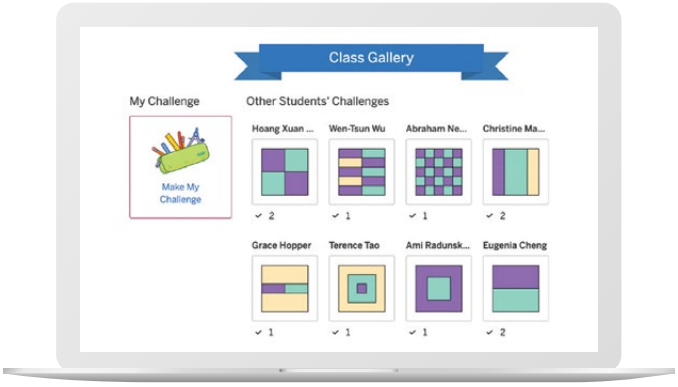
Procedural fluency is one of the five strands of mathematical proficiency described by the National Research Council, and it refers to the ability to carry out procedures flexibly, accurately, and efficiently. When students have procedural fluency, they select the appropriate strategies to use, solve problems in a reasonable amount of time, adapt strategies to apply them to new problem types, complete steps accurately, and answer problems correctly (Bay-Williams & SanGiovanni, 2021).

Every lesson in Amplify Desmos Math includes activities intended to develop students' procedural fluency, provide spiral review, and prepare them for summative assessments. Additionally, some lessons include repeated practice challenges, which encourage students to engage with mathematics through Responsive Feedback (see page 23) and collaboration with other students. This instructional approach gives educators opportunities to develop students' conceptual understanding through problem-solving while also providing them ample time to build procedural fluency (Chase, et al., 2009).

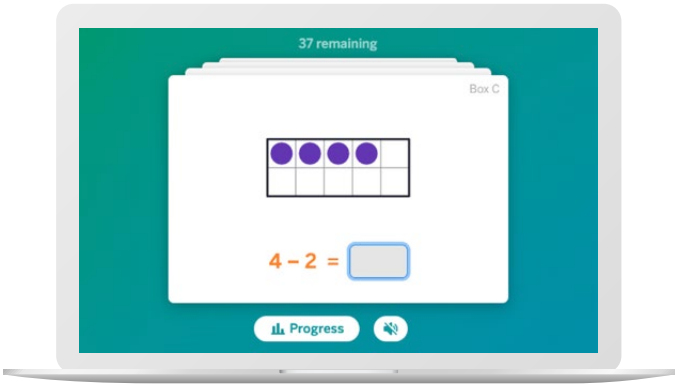
> "Procedural fluency is an essential component of equitable teaching and is necessary to developing mathematical proficiency and mathematical agency. Each and every student must have access to teaching that connects concepts to procedures, explicitly develops a reasonable repertoire of strategies and algorithms, provides substantial opportunities for students to learn to choose from among the strategies and algorithms in their repertoire, and implements assessment practices that attend to all components of fluency." (NCTM, 2023)

While conceptual understanding often supports the development of procedural knowledge, the relationship between conceptual and procedural understanding is often bidirectional (Rittle-Johnson et al., 2015). For example, when students have a strong number sense and can work with numbers flexibly, they can make sense of and use procedures meaningfully and appropriately when engaging with problems (Hunker & Bill, 2017). Through the process of practice and reinforcement, students are able to gain confidence in their ability to adapt their strategies to new problems.

The following are a few of the places students develop strong fluency within Amplify Desmos Math:



Challenge Creator supports students in connecting conceptual understanding and procedural fluency by prompting them to create their own challenge, then solve challenges created by classmates.



Fluency Practice provides adaptive, personalized practice with key math fluencies, including fact families, mental calculation, and number sense of whole numbers and fractions

Modeling and real-world applications

In order to apply math in context, students must make sense of quantitative relationships and use their mathematical toolbox to find unknown information or gain deeper insight into the situation at hand. A problem-based lesson structure supports this work by enabling students to compare and contrast their assumptions, mathematical models, and conclusions. Students are motivated to learn math by seeing how it can be used to analyze realistic, interesting, and relevant situations, and by actively participating in finding solutions (Chaseling et al., 2014; Gunderson et al., 2017).

Math that motivates

Amplify Desmos Math emphasizes social learning experiences that put student ideas at the center, helping students see themselves and their classmates as having powerful mathematical ideas. The curriculum includes countless opportunities for collaboration and hands-on, curiosity-driven learning, helping students dive into problems on their own and develop the skills of expressing their mathematical thinking and understanding the thinking of others. Amplify Desmos Math classrooms invite mathematical creativity, evoke wonder, and empower students to see themselves and their classmates as brilliant mathematicians.

Engagement and motivation

Research demonstrates that more engaged students exhibit more effort, display stronger coping skills and persistence, experience more positive emotions, and demonstrate increased attention in the classroom compared to their less-engaged peers (Fredricks et al., 2004; Lei et al., 2018; Skinner et al., 2016). Student engagement—encompassing behavioral, emotional, and cognitive involvement in academic pursuits—is a key construct in motivation research (Thijs & Verkuyten, 2009; Wong et al., 2024), and is fundamental to improving learning outcomes for students (Schlechty, 2001; Woolfolk & Margetts, 2007).

The Self-Determination Theory of Motivation provides valuable insight into fostering student engagement. It posits that students achieve optimal engagement and achievement when their needs for competence, autonomy, and relatedness are met within their learning environment. Competence is met in structured settings with clear criteria for success; autonomy, from the freedom to make choices; and relatedness, from emotionally supportive and socially inclusive learning environments (Ryan & Deci, 2000).

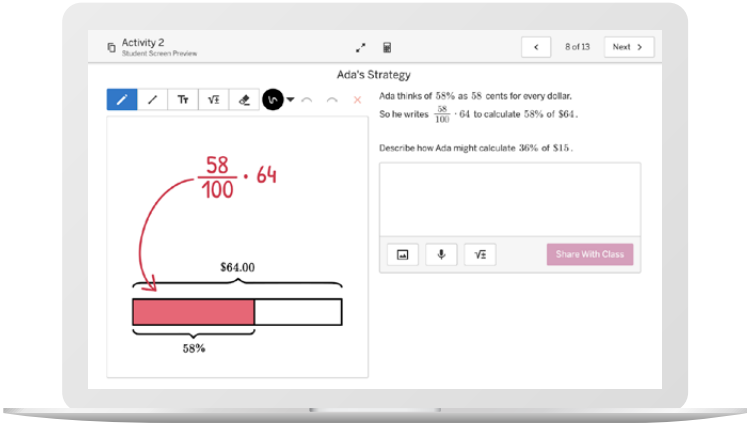
Research also underscores that student motivation is not solely an individual trait but rather emerges through dynamic interactions between the learner and their environment (Li & Sue, 2023). Horn (2017) outlines five factors that characterize motivational math classrooms:

- Belongingness comes from students' frequent, pleasant interactions with their peers and teachers.
- Meaningfulness answers the question, "When are we going to use this?"
- Competence helps all students discover their mathematical strengths.
- Accountability inspires students to participate in classroom life.
- Autonomy provides learners with tools for making sense of their work and seeing it through.

Amplify Desmos Math is designed to support educators in fostering a classroom culture where students want to join in and take risks. Each lesson inspires student motivation by providing multiple entry points to every activity and ensuring all students are supported and challenged. This asset-based approach helps teachers diagnose and build on what students already know, foster deep investment in math, and build agency. Amplify Desmos Math promotes a collaborative classroom where teachers are supported with explicit guidance in facilitating discussion, as well as what to look for and how to respond. Students are connected to each other's ideas and thinking, fueling classroom conversations and a shared understanding of math.

Fostering math identity

Math identity is a student's belief in their capability of doing math, and the value they place on math. Developing a strong math identity is pivotal for students' academic success. As K. Barba (2022) states, "Teaching mathematics is more than just the dissemination of content and the development of mathematical skills. It is also about empowering students to see themselves as participants and doers of mathematics." Developing a positive math identity transforms students from passive receivers to active participants in mathematical discourse and problem solving. Students with a strong math identity exhibit greater resilience when faced with complex problems and are more likely to persist, showing higher engagement and displaying a growth mindset. In fact, an adolescent's positive math identity is an important predictor of their postsecondary pursuits (Shifrer et al., 2023) and therefore extends far beyond the classroom.

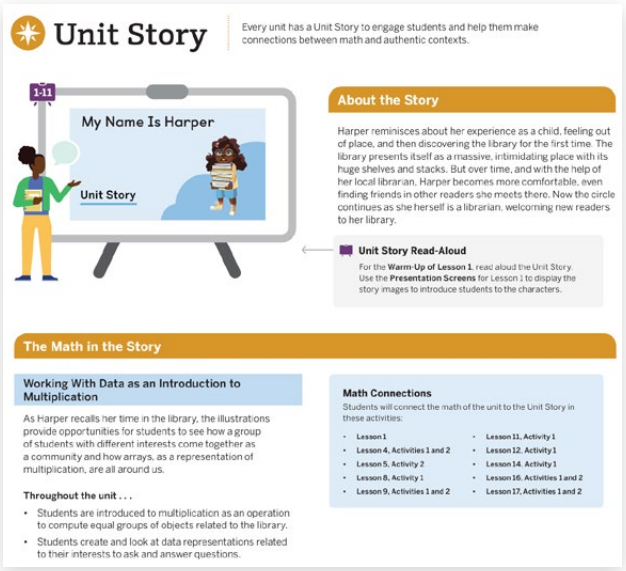


The classroom environment significantly influences the formation of math identity, and a teacher's pedagogical strategies play a critical role. Educators can foster a positive math identity by employing real-world problems and open-ended tasks to underline the relevance of math in everyday life. Martin (2006) found that teachers' enjoyment and confidence in teaching math, pedagogical efficacy, and emotional connection in the classroom have a positive impact on student engagement and motivation.

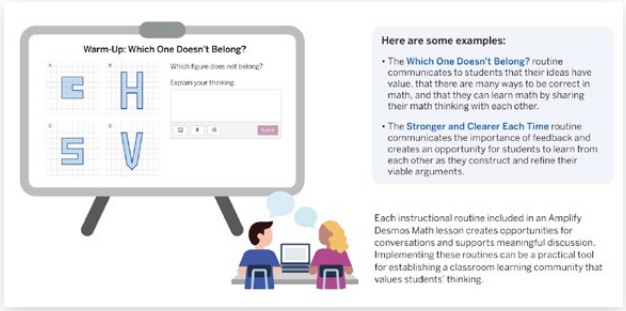
For both teachers and students, a pathway toward a positive math identity lies in developing a growth mindset: the belief that intellectual abilities are not fixed, but instead can be developed through effort, practice, and instruction. The importance of having a growth mindset has garnered significant attention in educational contexts, as research has shown that having a growth mindset predicts better academic performance (Blackwell, et al., 2007; Claro, et al., 2016; Cury, et al., 2006; Mangels, et al., 2006; Dweck, 2006).

Amplify Desmos Math helps bring math to life, developing positive student math identity with:

Unit Stories (K—5): Brief fiction stories read aloud by the teacher at the beginning of each unit that introduce contexts and characters connected to the math of the unit. Unit Stories help students see themselves and their communities in math.



Instructional Routines: Each routine (included in every lesson) creates opportunities for conversations and supports meaningful discussion. The implementation of these practical tools can help establish a classroom learning community that values students' thinking. (See page 35 for more on Math Language Routines.)



Amplify Desmos Math lessons are structured to elicit powerful math ideas from all students. The Math Identity and Community feature supports teachers in helping students build confidence in their own mathematical thinking, develop skills to work with and learn from others when doing math, and learn how math is an interwoven part of their broader community.

Math Identity and Community
Celebrate the variety of ways that students describe the same situation, highlighting how the diversity helps us have a broader understanding.

Math Identity and Community
Consider naming strategies after students in your class who share them. Encourage students to use each other's strategies by name throughout the lesson.

At Amplify Desmos Math, we believe in the brilliance of all students. By leveraging real-world problems and stories, Amplify Desmos Math helps teachers create connections, build positive math identities, and foster an engaged and collaborative math community with students at the center.

Being a student necessarily means having a lot to learn, but students aren't reducible to a list of the things they don't yet know. All students have strengths that can fuel their learning: They may already know some mathematics relevant to the lesson at hand, or they may have thought about a given type of question while at home or when making a purchase. All students bring knowledge from their homes and communities into the classroom every day. A curriculum can therefore develop student conceptual understanding (see page 12) through rich, real-world tasks that derive meaning from students' own experiences with them.

With Amplify Desmos Math, we focused on developing an engaging curriculum offering rigorous learning experiences that reflect the histories, cultures, and identities students bring to the classroom. We used Universal Design for Learning principles (see page 25) to design lessons that respect students' different approaches to problem solving and to ensure that the platform is intuitive and easy to use for all learners. The problem-based lessons and teacher facilitation tools encourage teachers to acknowledge students' range of thought and invite students to share their perspectives in making sense of contexts.

Amplify Desmos Math is built to support all students in accessing and participating in meaningful and challenging learning. This support is incorporated into the curriculum structure, lesson-level guidance, and digital tools.

Lesson facilitation supports
Every lesson includes at least one specific suggestion the teacher can use to increase access to the lesson without reducing the mathematical demand of the tasks. These suggestions address the following areas:

- Conceptual processing
- Visual-spatial processing
- Executive functioning
- Memory and attention
- Fine motor skills

Accessibility tools
Students have the ability to control accessibility tools so that each learning experience is customized to their individual needs. In many instances, these tools can be turned on or off at any point of instruction.

- **Text to speech:** Reads text instructions to students in multiple languages.
- **Enlarged font:** Increases the size of all text on screen.
- **Braille mode:** Includes narration of digital interactions.
- **Language selection:** Toggles between languages.

Student thinking is valuable and can be made evident.

Education aims to build students' critical thinking and reasoning, and mathematics education is no exception. Indeed, cognitive skills like these can predict the development of mathematical skill (Bellon et al., 2019). And metacognition, or knowledge of one's own cognitive processes and the ability to regulate these processes (Brown, 1978; Flavell, 1976), is a strong predictor of mathematical performance across domains, with robust effects in students from preschool through high school (Muncer et al., 2022; Ohtani & Hisasaka, 2018). Moreover, metacognition uniquely predicts student engagement in math over time (Wang et al., 2021).

Metacognition is a valuable window into student thinking, and it can take three forms (The University of Texas at Austin Center for Teaching and Learning, 2024):

- **Declarative knowledge:** Knowledge about one's self as a learner.
- **Procedural knowledge:** Knowledge about how to do things.
- **Conditional knowledge:** Knowledge about when and in what conditions certain knowledge is useful.

By illuminating how students are thinking about thinking, Amplify Desmos Math empowers students to see themselves as mathematicians and helps them build math proficiency for life. In our lessons, students take an active role in developing their own ideas first, then synthesize them as a class. The Amplify Desmos Math curriculum facilitates connections between students—through rich discourse, they learn from and engage with each other's thinking, and connect to each other through the shared understanding that they can use math to make sense of the world. This emphasis on the development of metacognition both motivates and engages students—and it can be developed through powerful, asset-based feedback.

Asset-based formative feedback is information communicated to students that is intended to modify their thinking for the purpose of improving learning. When studied, more thoughtful feedback produced larger effect sizes than verification or corrective feedback and was more effective for promoting higher-order learning outcomes. This was true for all participants, but with a greater effect in mathematics classrooms (Shute, 2008). Types of this feedback include the following:

- **Verification feedback:** Confirms whether an answer is correct or incorrect.
- **Corrective feedback:** Identifies the correct answer to a problem with no additional information.
- **Elaborated feedback:** Addresses the topic or response, discusses the particular error(s), provides worked examples, or gives supportive guidance.

Consequently, Amplify Desmos Math is built around the principles that feedback should:

- Address specific features of student responses in relation to the task, with suggestions for how to improve.
- Describe the what, how, and why of a given task.
- Be broken down into smaller, more manageable pieces so that it is not overwhelming and discarded, or does not invoke cognitive overload.
- Link clearly and specifically to goals and performance.
- Reinforce effort as a way to increase learning.
- Recognize mistakes as learning opportunities.

Responsive Feedback™

The delivery of feedback can have a pronounced impact on student engagement and learning outcomes. When they don't immediately know how to solve the problem, the implicit feedback they receive may convey a message of failure (when in actuality, the students are learning about the problem precisely because they don't yet know how to solve it). This is why Amplify Desmos Math prompts both students and teachers to reflect on their mathematical experiences, enabling students to produce creative and interesting representations of their thinking. The program asks questions that may have many interesting correct solutions, where even wrong answers offer the class something interesting to think about. This rough-draft thinking can then serve as the basis of a productive mathematical conversation.

> "Feedback is more effective the more information it contains. Simple forms of reinforcement and punishment have low effects, while high-information feedback is most effective." (Benedikt et al., 2020)

In every lesson, Amplify Desmos Math strives to:

- Invite students to share what they think can be learned from their peers' correct and incorrect thinking, helping students see and share the correct mathematical thinking behind every response and strategy.
- Celebrate variety in students' sketches and explanations.
- Help students see that mistakes are a normal part of learning and can be instructive.

The program uses what we call Responsive Feedback™ to visualize student ideas, helping them see the meaning behind their thinking. This motivates students and engages them in the learning process.

Productive struggle

> "Productive struggle is essential in mathematics. It encourages students to persist in working through challenging problems while developing problem-solving strategies, resilience, and a deeper understanding of mathematical concepts." (Hiebert & Grouws, 2007)

It can be challenging for teachers to watch their students struggle. Sometimes, struggle is perceived negatively or avoided altogether through immediate correction or help provided by teachers or peers. But struggle is an important part of the learning process (Steele et al., 2017). Teachers who support students through these moments of productive struggle are maintaining both instructional rigor and student engagement (Warshauer, 2015). According to NCTM (2014), "Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and support to engage in productive struggle as they grapple with mathematical ideas and relationships."

Productive struggle involves using existing knowledge to engage with challenging problems and persevering in problem solving (Baker et al., 2020). This level of rigor means having high expectations for all students, while also providing them with appropriate support (Blackburn, 2018). Through productive struggle, students build deeper conceptual understanding, thus enhancing their long-term memory and transfer of knowledge.

Productive struggle also communicates to students that learning is a process, that mistakes can become learning opportunities, and that persistence can be developed. This approach can transform student attitudes toward mathematics.

To support productive struggle in the mathematics classroom, teachers should carefully select problems that are within their students' zones of proximal development (Vygotsky, 1978): problems that are challenging but achievable with effort and persistence. Teachers should then provide support such as leading questions that stimulate thinking, but without short-circuiting the learning by reciting finished knowledge too soon (Cohen, 2011). (See pages 31 and 32 for more on differentiation and intervention.)

Teachers can support students through productive struggle by:

- Exploring the details of students' existing strategies.
- Providing students with time to think.
- Acknowledging students' struggles.

Amplify Desmos Math is designed to provide students with opportunities to grapple with mathematical ideas and relationships both individually and as a group. This prepares them for real-world mathematical challenges (and even non-mathematical challenges!) where patience, persistence, and creative problem-solving are key

to success. Tasks in the program are complex, with multiple entry points, and are structured to encourage collaboration that promotes persistence during problem solving. Teachers receive guidance on pacing, feedback, and effective questioning strategies to support learning when students feel stuck. Students are empowered to take risks, and develop cognitive routines that promote persistence in problem-solving.

Classroom discourse

Mathematical discourse is associated with increased learning gains for students (e.g., Carpenter et al., 2003; O'Connor, 2010; Smith & Stein, 2011). The positioning of students in the classroom, the ways a curriculum elicits their thinking, and the use of conversation strategies are central to discourse-rich classrooms and important for providing access for all students (Hunker & Bill, 2017). Recent studies (Smith & Stein, 2011) have shown that students learn mathematics best when they have opportunities to:

- Explain and justify solution strategies.
- Construct arguments.
- Pose questions.
- Critique the reasoning of others.
- Make connections between ideas.

In doing so, students learn from one another by hearing ideas that are different from their own, articulating their understanding to others, and collectively gaining knowledge that goes beyond any one individual's understanding (Staples & King, 2017).

Valuing student ideas and strategies, emphasizing sense-making, and encouraging reasoning through discourse can foster an equitable learning environment (Chapin & O'Connor, 2013). Classrooms with strong discourse practices honor all student voices (Aguirre et al., 2013), including the voices of students who work at different paces or have varying depths of understanding (Huinker & Bill, 2017). This collaborative approach not only fosters a sense of community within the classroom, but promotes active engagement, helps students develop confidence in their abilities, and encourages students to take ownership of their learning (Staples & King, 2017). In this environment, students begin to see themselves as thinkers and doers of mathematics.

Planning for mathematical discourse is important. Teachers can strategically prepare for and implement certain strategies when facilitating mathematical discourse in their classrooms, like anticipating student thinking, posing purposeful questions, or facilitating whole-group discussion (Smith & Stein, 2011).

Anticipating what students understand, how they will interpret a problem, and what strategies they might use to solve it is an integral part of orchestrating productive mathematics discourse (Smith & Stein, 2018). By listening for students' mathematical thinking (not just their answers to problems), teachers can focus on fostering a deeper understanding of mathematics. In addition to being pedagogically sound, this approach also contributes to inclusivity and access for all students, while enabling teachers to design lessons that build on students' existing knowledge, value different ways of thinking, and provide appropriate challenges. Planning with colleagues can further enhance this process. Together, teachers can discuss how they will monitor student work, formulate questions based on specific strategies, and determine key points to support students throughout a lesson (Smith & Stein, 2011; Vale et al., 2019).

To strengthen, clarify, and extend students' thinking throughout a lesson, it's important that teachers plan for and pose purposeful questions (NCTM, 2014). The art of questioning is "a powerful tool to support students in their building of mathematical ideas" (Martino & Maher, 1999). Moreover, the types of questions teachers ask influence the learning that occurs. For instance, closed-ended questions that elicit a single-word response afford different learning opportunities than questions that encourage students to engage in high levels of reasoning. Therefore, posing purposeful questions is an essential teaching practice that effectively fosters a clear understanding of mathematical concepts and promotes active student engagement in the learning process.

Amplify Desmos Math anticipates all of the different ways a student might solve a given task in a lesson, approaching each with the following questions:

- What strategies might a student use?
- What are common misconceptions students might have?
- What other assets do students bring into this lesson?
- How can we further extend/challenge students?

Effective whole-group discussions provide opportunities for students to share their ideas, engage in conversations with their peers, and advance their understanding of key mathematics ideas (Staples & King, 2017). Encouraging students to engage in quantitative reasoning, construct explanations, and critique the reasoning of others supports the development of their understanding of critical mathematical concepts and relationships. Students need multiple opportunities to talk about their thinking and collaborate with peers to solve complex problems, with the support of targeted guidance from the teacher (Cazden, 2001; Moschkovich, 2013). This requires both building on students' thinking and guiding them toward an understanding of key mathematics concepts and skills, a balance that necessitates important in-the-moment decision-making (Herbel-Eisenmann et al., 2011; Kooloos et al., 2022). This is particularly important during the lesson synthesis, which is where learning unfolds within the whole classroom group, and why careful planning and adequate

time allocation are critical (Van de Walle et al., 2018). Amplify Desmos Math lessons provide guidance for teachers facilitating these crucial discussions, giving them the support they need to help students make sense of mathematical ideas, debate strategies, make connections, and find patterns in the math (Smith et al., 2008).

When facilitating discourse, Amplify Desmos Math draws on Stanford's Principles for the Design of Mathematics Curricula (Zwiers et al., 2017), which provides a design framework to promote equity in mathematical language use and development (Dieckmann et al., 2019). These principles were designed to promote the following four key functions (Dieckmann et al., 2019):

- **Support sense-making:** Routines should help students make sense of the language, the skills, and the math.
- **Optimize output:** Routines should help students get progressively better at expressing their thinking when writing and speaking about math.
- **Cultivate conversation:** Routines should help students have rich interactions to help fill in knowledge gaps, make mistakes, and collectively correct mistakes. They should also provide a low-stakes way for students to strengthen mathematical thinking and language use.
- **Maximize meta-awareness:** Routines should help students organize thinking, apply concepts across contexts, and reflect on their use of math language.

This framework outlines key Math Language Routines for educators to implement with students. The following are some of the routines included in Amplify Desmos Math that promote discourse:

- Co-craft Questions
- Collect and Display
- Compare and Connect
- Critique, Correct, Clarify
- Decide and Defend
- Notice and Wonder
- Number Talk
- Stronger and Clearer Each Time
- Tell a Story
- Think-Pair-Share
- Three Reads
- Which One Doesn't Belong?

Access to grade-level understanding for every student, every day

To ensure all students are able to access grade-level math, Amplify Desmos Math includes core instruction, a suite of assessments, and tailored personalized learning resources that adjust to meet student needs. Multiple points of entry to lesson content and Responsive Feedback ensure that every student feels challenged and successful. Cohesive differentiation and intervention resources provide the necessary support to prevent students from falling behind and empower them to extend their thinking.

The asset-based assessment system of Amplify Desmos Math not only evaluates what students know about grade-level math, but also provides insight into how they think. The reliable measures of mCLASS® Assessments analyze student responses to reveal underlying mathematical thinking and empower teachers with targeted, actionable insights and supplemental intervention resources linked to core instruction.

SCREENING AND PROGRESS MONITORING

mCLASS Assessments

Measure what students know and how they think.

CORE INSTRUCTION

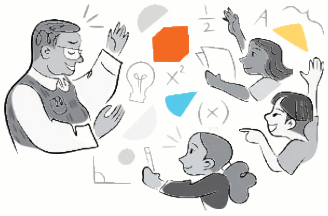
Amplify Desmos Math

Develop lasting grade-level understanding.

DIFFERENTIATION AND INTERVENTION

Targeted Teaching Tools

Boost Personalized Learning



Small group instruction, skill practice, and extension activities

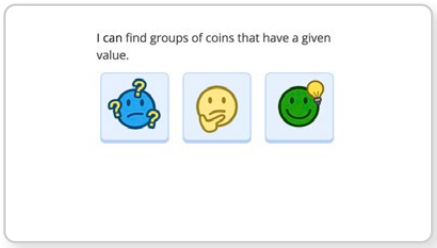
Universal Design for Learning

The Universal Design for Learning (UDL) framework posits that educational practices must be flexible to support all students' learning needs (CAST, 2018). Designed to foster an accessible and inclusive learning community, the UDL framework is crucial in all classrooms—but especially in those with a diverse group of students. The following set of three UDL principles (Rose & Meyer, 2002) helps educators integrate flexible options into their curriculum and instruction (Rao & Meo, 2016):

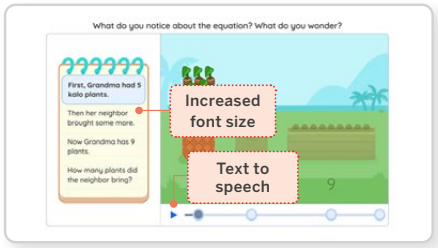
Multiple Means of Engagement
Individuals are motivated in different ways, at different times, and in different contexts. Lessons provide learners a variety of options to recruit their interest, support sustained effort and persistence, and promote self-regulation.

Multiple Means of Representation
Learners make sense of information differently. Amplify Desmos Math includes options for presenting information in multiple ways to support comprehension and understanding of language.

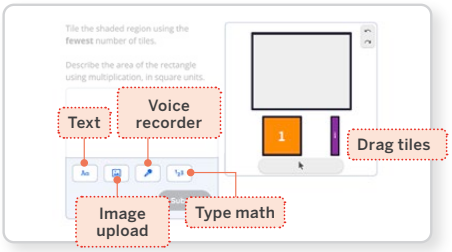
Multiple Means of Action and Expression
Learners differ in how they navigate learning environments and express what they know. Amplify Desmos Math ensures that materials are accessible, support multiple means of students' expression and communication, and scaffold executive functioning.



Students reflect on their learning.



Students and teachers can use digital tools, such as text to speech, enlarged font, and dynamic visuals.



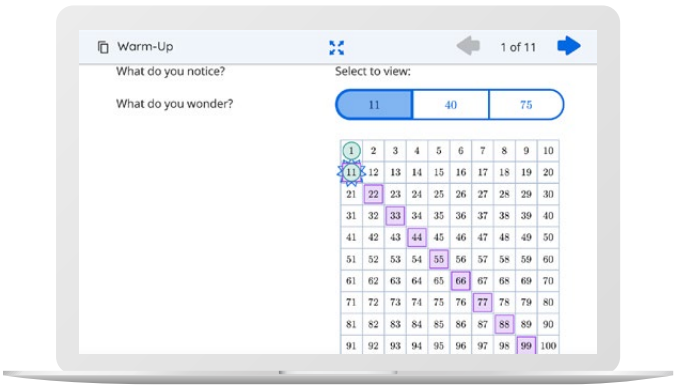
Students can communicate their ideas in multiple ways, including sketching, uploading photos, or recording an audio response.

In alignment with these principles, Amplify Desmos Math presents information in multiple forms in order to cater to the learning modalities that are optimal for each student. Each unit of Amplify Desmos Math activates students' previously learned content, prior knowledge, and perspectives from their real-world lived experiences, thus maximizing the transfer of knowledge to new concepts and situations.

The UDL framework also emphasizes the need for students to interact with learning materials and express what they've learned in different ways. Amplify Desmos Math takes this idea and builds on it by incorporating various assessment methods and technological tools, moving beyond traditional paper-and-pencil tasks to incorporate hands-on manipulatives, visually-engaging digital activities, teacher Presentation Screens, and student discourse throughout lessons. Additionally, Amplify Desmos Math gives students regular opportunities for self-assessment by providing actionable feedback that promotes extended learning. (See page 20 for more on Responsive Feedback.)

The UDL principles encourage curriculum and instruction to prioritize student engagement by taking students' interests, motivations, and persistence into consideration. Contextualizing targeted skills in real-world applications optimizes relevance, value, and authenticity for students (Root et al., 2019), which is why Amplify Desmos Math grounds every unit in real-world problems. And because UDL tasks are designed with all learners in mind, Amplify Desmos Math lessons always begin with low-floor, high-ceiling warm-up activities.

Low-floor, high-ceiling tasks: These math activities allow all group members to start and work at their own level, with the possibility of taking on more challenging tasks as the activity progresses.



Engage all students by providing an easy entry level (low floor) while also offering extensions that challenge students (high ceiling) (Bobis et al., 2021). These tasks encourage students to struggle with important mathematical ideas (Hiebert & Grouws, 2007) and experience some level of confusion (Bobis et al., 2021). As students use a variety of strategies to generate their own solutions, they develop a deeper conceptual understanding (Russo et al., 2020; Sullivan et al., 2015).

The UDL principles also underline the importance of fostering peer learning and building resilience. Following these guidelines, Amplify Desmos Math promotes inclusive instruction by treating moments of productive struggle as instructional opportunities and by encouraging students to collaborate and learn from one another. These practices are built into every lesson of the program. Amplify Desmos Math also provides multiple entry points and multiple learning modalities across the program. Lessons engage students with both digital and physical resources (like paper, pencils, and other manipulatives).

While creating Amplify Desmos Math, we asked ourselves the following questions about each and every activity:

- Should students engage with this on paper or on a computer?
- How should students show their thinking? Should we ask students to type a text response into the computer, or ask them to discuss their thinking with a partner?
- What early experiences, personal intuitions, or cultural knowledge could students make use of in this lesson?
- How can we engage all learners and get students out of their desks and moving?
- How can every student access this content?

Helping students learn math and love learning math is a demanding—but not impossible—task. Students are the most underutilized resource in a classroom (Confrey, 2014), which is why we focus on connecting them to each other, creating opportunities for conversation, and bringing their experiences and strengths into the math classroom. Each lesson provides opportunities for students to collaborate, pairing rich and authentic tasks with structured peer support. Educators can make content more accessible, as well as more culturally and personally relevant to students, by embracing the social aspect of learning and leveraging the rich cultural backgrounds and experiences that students bring into the learning environment (NCTM, 2000; Gutiérrez, 2011). Collaborative learning environments (where students engage in appropriately challenging tasks with their peers) not only render learning goals more attainable, but also contribute to a positive and motivating classroom climate (Bandura, 1986; Mueller et al., 2011; Urdan & Turner, 2005).

Multilingual/English learners

Multilingualism is an asset, and Amplify Desmos Math leverages it to support multilingual/English learners (ML/ELs) in the classroom. The program is designed to bring together multilingual instructional best practices and research findings in ways that fill the gaps left by other deficit-based curricula (de Araujo & Smith, 2022).

Mathematical language development is critical for all students, not just ML/ELs. Working in close collaboration with the English Learner Success Forum, the Amplify Desmos Math team has created an integrated, comprehensive strategy to support both ML/ELs and the entire classroom in developing mathematical language.

The Amplify Desmos Math approach to math language development focuses on when, how, and why students are using language to make sense of and share their mathematical ideas. Every lesson in Amplify Desmos Math includes opportunities for all students to develop mathematical language as they experience the content, while simultaneously providing intentional support for ML/ELs. We purposefully progress language throughout the units by cultivating students' language development and supporting students in making their arguments and explanations stronger, clearer, and more precise as they progress. This work also stems from our commitment to support the continued development of positive math identity and community.

Vocabulary

Units and lessons start by surfacing students' language for new concepts and then build connections between their language and the new vocabulary for that unit. This leverages the language assets that students bring into their learning.

Math Language Routines (MLR)

Math Language Routines[1] are used within lessons to do one or more of the following: highlight student-developed language and ideas, cultivate conversation, support mathematical sense-making, and promote meta-cognition. Tips for facilitating MLRs are included when they would be helpful within lessons.

Language goals

Language goals attend to the mathematics students are learning and are written through the lens of one or more of four language modalities: **Reading**, **Writing**, **Speaking**, and **Listening**.

Multilingual/English learner support

Specific, targeted supports that would be beneficial to ML/ELs are suggested at intentional points within each lesson. They often describe a modification to increase access to the task or support with contextual or mathematical language development that can often be supportive of all learners. ML/EL supports may also be attached to MLRs.

Spanish translations of student-facing print and digital materials will be available for the 2025–26 school year.

Language plays an important role in mathematics, as it not only facilitates students' communication of mathematical ideas clearly and accurately, but also supports the development of conceptual understanding, problem-solving skills, and the ability to engage in meaningful mathematical discourse. Strengthening language is critical for promoting students' mathematical learning (Erath et al., 2021). That's why Amplify Desmos Math lessons provide ample opportunities for ML/ELs to explain their thinking and master academic language (Merritt et al., 2016).

Effective mathematics learning for ML/ELs begins with teachers serving as mentors and guiding students in mastering mathematical language through everyday practice. In developing Amplify Desmos Math, we utilized a set of instructional practices teachers can adopt to support ML/ELs that include opportunities for students to explain and clarify their thinking with others, draw on various linguistic and nonlinguistic resources, respond to teacher discourse moves, and solve relevant and familiar math tasks (Maldonado et al., 2010). Other research points to the importance of employing multimodal representations in instruction, reinforcing key vocabulary, and defining and analyzing errors (Merritt et al., 2016). Additionally, encouraging students to engage in mathematical discourse promotes usage—and, therefore, understanding—of math language by giving students opportunities to organize and communicate their thoughts, analyze strategies, and justify solutions using mathematical language (Maldonado et al., 2010; Moschkovich, 1999; NCTM, 2014; Sacco et al., 2022).

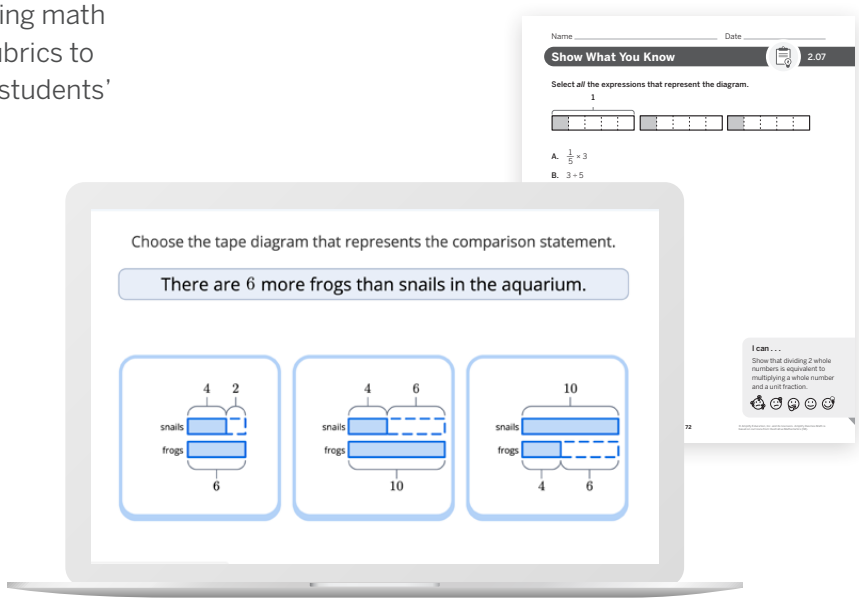
To meet the needs of students, Amplify Desmos Math ties together multiple best practices that serve not only ML/ELs but the entire class. These include connecting math to real-life experiences, framing math as a community activity, using language to guide lessons, and teaching key vocabulary in ways that promote long-term memory (Torres-Velasquez & Lobo, 2005). Amplify Desmos Math views ML/ELs in terms of their existing skills instead of their English proficiency, and views their native language and experiences as assets instead of obstacles (Moschkovich, 2007, 2011).

Multi-Tiered System of Supports

> "[An] MTSS is a proactive and preventative framework that integrates data and instruction to maximize student achievement and support students from a strengths-based perspective." (American Institutes for Research, 2024)

Amplify Desmos Math follows a Multi-Tiered System of Supports (MTSS) framework. This proactive, preventative approach to intervention emphasizes data-based decision-making that responds to students' needs with research-backed, high-quality instruction and intervention. Amplify Desmos Math is supported by the following MTSS:

- **mCLASS Benchmark*:** A universal screener that evaluates student performance toward grade-level expectations for growth and identifies specific strengths and development needs for each student to inform differentiation in Amplify Desmos Math and Boost Personalized Learning
- **mCLASS Progress Monitoring*:** An assessment that helps teachers evaluate student progress between benchmark assessment periods and informs whether an intervention is working or whether adjustments are needed to improve student learning
- **Unit-level assessments:** Embedded unit assessments that offer key insights into students' conceptual understanding of math by providing regular, actionable information about how students are thinking about and processing math (using both auto-scoring and in-depth rubrics to help teachers anticipate and respond to students' learning needs)
- **Show What You Know Assessments:** Daily formative assessments (Wiliam, 2017) focused on one of the key concepts in a lesson that give students opportunities to show their teacher what they understand and what they are still learning
- **Reporting:** Integrated unit assessments, lesson assessments, Boost Personalized Learning, benchmark assessments, and progress monitoring reports that provide a comprehensive look at student learning and enable teachers to make data-based decisions



*Feature currently supports grades K–8.

Research has consistently found that children's early mathematics knowledge predicts their later mathematics achievement (Nelson & Powell, 2018; Nguyen et al., 2016; Rittle-Johnson et al., 2017; Watts et al., 2014). This finding underscores the importance of implementing interventions or preventative measures early and in classrooms to mitigate academic challenges and prevent students from falling behind (Walker & Shinn, 2010).

The most substantial positive effects of intervention and preventative measures occur when they are directly aligned with the instructional and curricular goals of the classroom (Gersten, 2016). Implementing tailored, just-in-time intervention and instruction better supports students' academic growth, and interventions introduced at lower levels reduce the need for interventions at subsequent levels (Walker & Shinn, 2010). Additionally, it is important that interventions are engaging and highlight students' academic, cultural, and motivational assets.

Amplify Desmos Math is grounded in the idea that the best kind of intervention is prevention, and that this prevention is key to implementing a multi-tiered approach. Through the use of high-quality instructional materials and systematic, explicit instruction for all students (with increasing intensity for those identified as being at risk), this robust system ensures access to grade-level content for all students.

Intervention

Amplify Desmos Math embeds 15-minute Mini-Lessons throughout each unit, each of which is aligned to the most critical topics within that unit. These targeted, small-group interventions can be leveraged to address prerequisite skills, or to re-engage students in grade-level math based on performance during core lessons, mCLASS Math Assessments, and other Boost Personalized Learning activities. The design behind this intervention is informed by the extensive research around worked examples.

Worked examples consist of problem formulation and an outline of the steps needed to reach a solution. Classroom discourse centered on worked examples serves to deepen understanding by directing students' focus toward problem-solving processes rather than mere application of memorized procedures. This approach supports the development of student reasoning, an improvement over the conventional practice of independent problem-solving with minimal support or feedback (Riccomini & Morano, 2019). Over time, this intervention helps students become independent problem solvers (Renkl et al., 2002) and promotes both near and far transfer of learning over time (Atkinson et al., 2003).

Differentiation

Each student comes to the classroom with unique strengths and patterns of thinking. This understanding forms the basis of differentiation, a research-based teaching approach that tailors core curriculum instruction to meet the needs of each individual student. Differentiation draws on students' assets to keep them engaged and motivated, while also increasing their autonomy (Tomlinson, 2014). By acknowledging and accommodating students' differences, we create an environment where all students can engage more deeply with the learning process, ultimately leading to greater academic growth.

Differentiation is an ongoing process where teachers actively seek out and respond to the diverse academic needs present in their classrooms (Bondie et al., 2019). This involves making adjustments to instruction to enhance Clarity, Access, Rigor, and Relevance (CARR) for every student within the learning community. In simpler terms, differentiation is about ensuring that all students have clear pathways to learning and opportunities to succeed.

Differentiation is about meeting students where they are academically, but it's also about recognizing and honoring the diverse range of strengths and needs they bring to the classroom (Pozas et al., 2019). Differentiation extends beyond academic differences to encompass variations in student motivation, interests, and background. Understanding and addressing these aspects is essential for creating inclusive learning environments where all students feel valued and empowered to succeed (Roy et al., 2013; Tomlinsson, 2014; Lindner & Schwab, 2020).

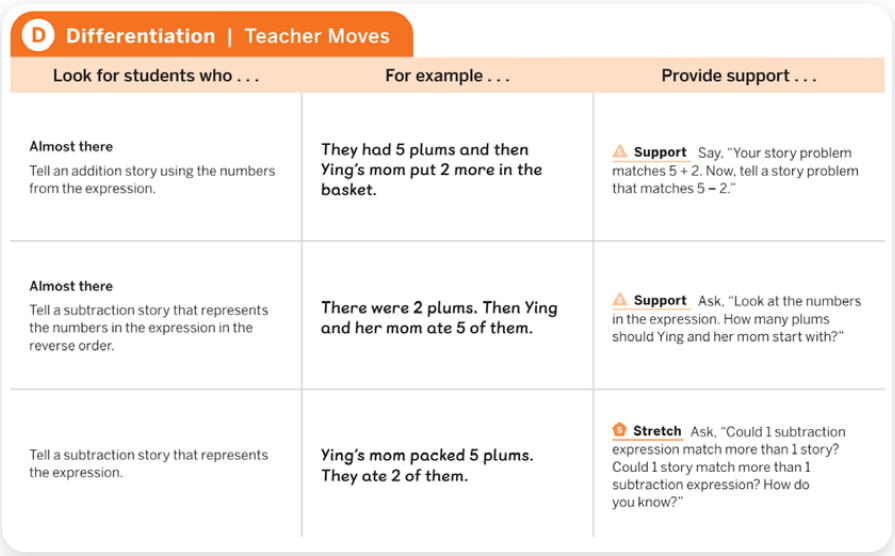
We built the Amplify Desmos Math suite of resources to be a truly cohesive experience for both teachers and students. Each part of the program—from beginning-of-year diagnostic screeners and daily core instruction to personalized practice and interim assessments—lives in the same ecosystem, communicates across the platform, and feeds into the same data sources.

Because K–12 math can be a gateway to future opportunities, it's of critical importance that we help students catch up while also keeping up with grade-level math. That's why Amplify Desmos Math provides recommendations when a student shows they could benefit from support to gain access to grade-level work. These can include teacher-led interventions, personalized practice, or both. And because every child deserves the opportunity to stretch to their fullest potential, our differentiation offers equitable access to enrichment opportunities, too.

In response to the shift in the educational landscape due to the COVID-19 pandemic, an opportunity has emerged to enhance math intervention strategies, empowering students to leverage their inherent strengths and potential. This empowerment leads them to transform their learning experiences, using challenges as catalysts for growth.

Practically speaking, differentiation encompasses a variety of strategies, including modifying the level of support provided through scaffolding, offering alternative ways for students to access content, and employing flexible grouping arrangements based on students' needs and abilities. It also involves continually monitoring student progress; utilizing adaptive technology; and adapting instruction to provide additional support, challenge, or intervention as necessary (Deunk et al., 2018).

Every lesson in Amplify Desmos Math includes differentiation recommendations for resources to support, strengthen, and stretch.



D Differentiation | Teacher Moves

Look for students who . . .	For example . . .	Provide support . . .
Almost there Tell an addition story using the numbers from the expression.	They had 5 plums and then Ying's mom put 2 more in the basket.	**Support** Say, "Your story problem matches 5 + 2. Now, tell a story problem that matches 5 – 2."
Almost there Tell a subtraction story that represents the numbers in the expression in the reverse order.	There were 2 plums. Then Ying and her mom ate 5 of them.	**Support** Ask, "Look at the numbers in the expression. How many plums should Ying and her mom start with?"
Tell a subtraction story that represents the expression.	Ying's mom packed 5 plums. They ate 2 of them.	**Stretch** Ask, "Could 1 subtraction expression match more than 1 story? Could 1 story match more than 1 subtraction expression? How do you know?"

Differentiation tables provide tips to support, strengthen, and stretch based on student needs.

Some differentiation resources include:

- **Boost Personalized Learning (K–8):** Engaging, independent digital practice that provides access to grade-level math through Responsive Feedback and adjusts to student work with item-level adaptivity to support learning
- **Centers (K–5):** Lesson-embedded routines and practice that are vertically aligned across grade levels
- **Math Adventures (K–8):** Strategy-based math games that enable students to explore mathematical concepts
- **Extensions:** Lesson-embedded teacher moves, including student-choice activities and stretch questions for students
- **Item Bank (K–8):** Space for teachers to create practice and customize assessment activities by using filters and searching for standards, summative-style items, and more

References

Adding it up: Helping children learn mathematics. (2001). National Academies Press. https://doi.org/10.17226/9822.

Aguirre, J., Mayfield-Ingram, K., & Martin, D. B. (2013). The impact of identity in K–8 mathematics learning and teaching: Rethinking equity-based practices. The National Council of Teachers of Mathematics.

Anderson, J. R., Fincham, J. M., & Douglass, S. (1997). The role of examples and rules in the acquisition of a cognitive skill. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 23*, 932–945.

Atkinson, R. K., Renkl, A., & Merrill, M. M. (2003). Transitioning from studying examples to solving problems: Effects of self-explanation prompts and fading worked-out steps. *Journal of Educational Psychology, 95*(4), 774–783. https://doi-org.ezproxy.cul.columbia.edu/10.1037/0022-0663.95.4.774

Avalos, M. A., Medina, E., & Cicada, W. G. (2015). Planning for instruction: Increasing multilingual learners' access to algebraic word problems and visual graphics. In A. Bright, H. Hansen-Thomas, & L. de Oliveira (Eds.), *The Common Core State Standards in Mathematics for English language learners, high school* (pp. 5–23). TESOL Press.

Baker, K., Jessup, N. A., Jacobs, V. R., Empson, S. B., & Case, J. (2020). Productive struggle in action. *Mathematics Teacher: Learning and Teaching PK–12, 113*(5), 361–367. https://doi.org/10.5951/mtlt.2019.0060

Barba, K. M. (2022). Mathematical identity and the role of the educator. *Journal of Mathematics Education at Teachers College, 13*(1), 7–13. https://doi.org/10.52214/jmetc.v13i1.9187

Battista, M. (2017). Mathematical Reasoning and Sense Making in Grades 6–8. NCTM.

Bellon, E., Fias, W., & De Smedt, B. (2019). More than number sense: The additional role of executive functions and metacognition in arithmetic. *Journal of Experimental Child Psychology, 182*, 38–60.

Blackburn, B. (2018). Productive struggle is a learner's sweet spot. ASCD. https://www.ascd.org/el/articles/productive-struggle-is-a-learners-sweet-spot

Blackwell, L. S., Trzesniewski, K. H., & Dweck, C. S. (2007). Implicit theories of intelligence predict achievement across an adolescent transition: A longitudinal study and an intervention. *Child Development, 78*, 246–263.

Bobis, J., Russo, J., Downton, A., Feng, M., Livy, S., McCormick, M., & Sullivan, P. (2021). Instructional moves that increase chances of engaging all students in learning mathematics. *Mathematics, 9*(6). 582.

Bondie, R. S., Dahnke, C., & Zusho, A. (2019). How does changing "one-size-fits-all" to differentiated instruction affect teaching? *Review of Research in Education, 43*(1), 336–362.

Brown, A. L. (1978). Knowing when, where, and how to remember: A problem of metacognition. *Advances in Instructional Psychology 1*, pp. 77–165.

CAST. (2024, April 3). CAST. https://www.cast.org/

Cazden, C. B. (2001) *Classroom discourse: The language of teaching and learning* (2nd ed.). Portsmouth: Heinemann.

Celedón-Pattichis, S., Borden, L. L., Pape, S. J., Clements, D. H., Peters, S. A., Males, J. R., Chapman, O., & Leonard, J. (2018). Asset-based approaches to equitable mathematics education research and practice. *Journal for Research in Mathematics Education, 49*(4), 373–389. https://doi.org/10.5951/jresematheduc.49.4.0373

Chase, C., Chin, D.B., Oppezzo, M., Schwartz, D.L. (2009). Teachable agents and the protégé effect: Increasing the effort towards learning. *Journal of Science Education and Technology, 18,* 334–352. https://doi.org/10.1007/s10956-009-9180-4

Claro, S., Paunesku, D., & Dweck, C. S. (2016). Growth mindset tempers the effects of poverty on academic achievement. Proceedings of the National Academy of Sciences, 113(31), 8664–8668.

Cohen, D. K. (2011). *Teaching and its predicaments*. Harvard University Press.

Cohen, D.K., Raudenbush, S., & Ball, D.L. (2003). Resources, instruction, and research. *Educational Evaluation and Policy Analysis, 25,* 119–142. https://doi.org/10.3102/01623737025002119

Confrey, J., Maloney, A. P., & Corley, A. K. (2014). Learning trajectories: A framework for connecting standards with curriculum. *ZDM, 46*(5), 719–733. https://doi.org/10.1007/s11858-014-0598-7

Cury, F., Elliot, A. J., Da Fonseca, D., & Moller, A. C. (2006). The social-cognitive model of achievement motivation and the 2× 2 achievement goal framework. *Journal of Personality and Social Psychology, 90*(4), 666.

de Araujo, Z., & Smith, E. (2022). Examining English language learners' learning needs through the lens of algebra curriculum materials. *Educational Studies in Mathematics, 109*, 65–87.

Deunk, M. I., Smale-Jacobse, A. E., de Boer, H., Doolaard, S., & Bosker, R. J. (2018). Effective differentiation practices: A systematic review and meta-analysis of studies on the cognitive effects of differentiation practices in primary education. *Educational Research Review, 24,* 31–54.

Dieckmann J., & Skarin, R. (2019). Developing the math language routines. Achieve the Core: Peers and Pedagogy. achievethecore.org/peersandpedagogy/developing-math-language-routines/

Driscoll, M. P. (1994). *Psychology of learning for instruction*. Allyn & Bacon.

Dweck, C. (2006). *Mindset: The new psychology of success*. Ballantine Books.

Dweck, C. S., & Leggett, E. L. (1988). A social-cognitive approach to motivation and personality. *Psychological Review, 95*(2), 256–273. https://doi.org/10.1037/0033-295x.95.2.256

Flavell, J. H. (1976). Metacognitive aspects of problem-solving. In L. B. Resnick (Ed.), *The nature of intelligence* (pp. 231–236). Erlbaum.

Fredricks, J. A., Blumenfeld, P. C., & Paris, A. H. (2004). School engagement: Potential of the concept, state of the evidence. *Review of Educational Research, 74*(1), 59–109. http://dx.doi.org/10.3102/00346543074001059

Gersten, R. (2016). What we are learning about mathematics interventions and conducting research on mathematics interventions. *Journal of Research on Educational Effectiveness, 9*(4), 684–688.

Gutiérrez, R. (2011). Context matters: How should we conceptualize equity in mathematics education? In B. Herbel-Eisenmann, J. Choppin, D. Wagner, & D. Pimm (Eds.), *Equity in discourse for mathematics education: Theories, practices, and policies* (pp. 17–33). Springer Netherlands.

Hausfather, S. J. (1996). Vygotsky and schooling: Creating a social contest for learning. *Action in Teacher Education,* (18),1–10.

Herbel-Eisenmann, B. A., Steele, M. D., & Cirillo, M. (2013). (Developing) teacher discourse moves: A framework for professional development. *Mathematics Teacher Educator, 1*(2), 181–196. https://doi.org/10.5951/mathteaceduc.1.2.0181

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester, Jr. (Ed.). *Second handbook of research on mathematics teaching and learning* (pp. 371–404). Information Age.

Hmelo-Silver, C. (2004). Problem-based learning: What and how do students learn? *Educational Psychology Review, 16*(3), 235-266.

Hmelo-Silver, C. E., & Barrows, H. S. (2006). Goals and strategies of a problem-based learning

facilitator. *The interdisciplinary Journal of Problem-based Learning, 1*(1), 21-39.

Horn, I. S. (2017). *Motivated: Designing math classrooms where students want to join in.* Heinemann.

Jackson, K. J., Shahan, E. C., Gibbons, L. K., & Cobb, P. A. (2012). Launching complex tasks. *Mathematics Teaching in the Middle School, 18*(1), 24–29. https://doi.org/10.5951/mathteacmiddscho.18.1.0024

Jaffe, E. (2020). Mindset in the classroom: Changing the way students see themselves in mathematics and beyond. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas, 93*(5), 255–263.

Kooloos, C., Oolbekkink-Marchand, H., van Boven, S., Kaenders, R., & Heckman, G. (2022). Making sense of student mathematical thinking: The role of teacher mathematical thinking. *Educational Studies in Mathematics, 110*(3), 503–524. https://doi.org/10.1007/s10649-021-10124-2

Lei, H., Cui, Y., & Zhou, W. (2018). Relationships between student engagement and academic achievement: A meta-analysis. *Social Behavior and Personality: An International Journal, 46*(3), 517–528. https://doi.org/10.2224/sbp.7054

Lester, Jr., F. K. (Ed.). (2003). *Teaching mathematics through problem solving*. National Council of Teachers of Mathematics.

Li, J., & Xue, E. (2023). Dynamic interaction between student learning behaviour and learning environment: Meta-analysis of student engagement and its influencing factors. *Behavioral Sciences, 13*(1), 59. https://doi.org/10.3390/bs13010059

Liljedahl, P., Wheeler, L., & Müller, M. (2020). *Denkklassen Creëren = Building Thinking Classrooms: 14 strategieën om leerlingen maximaal aan het denken te zetten bij rekenen/wiskunde.* Bazalt Groep.

Lindner, K.-T., & Schwab, S. (2020). Differentiation and individualisation in inclusive education: A systematic review and narrative synthesis. *International Journal of Inclusive Education,* 1–21. https://doi.org/10.1080/13603116.2020.1813450

López, F. A. (2017). Altering the trajectory of the self-fulfilling prophecy. *Journal of Teacher Education, 68*(2), 193–212. https://doi.org/10.1177/0022487116685751

Lubis, R. H., Syahputra, E., & Siagian, P. (2018). Improving students ability in mathematical problem solving through problem based learning model in tenth grades State Senior High School 7 of Medan [Conference session].

Maldonado, L. A., Turner, E. E., Dominguez, H., & Empson, S. B. (2010). English-language learners learning from, and contributing to, mathematical discussions. In D. Y. White & J. S. Spitzer (Eds.), *Mathematics for every student: Responding to diversity grades Pre-K–5* (pp. 7–22). National Council of Teachers of Mathematics.

Mangels, J. A., Butterfield, B., Lamb, J., Good, C., & Dweck, C. S. (2006). Why do beliefs about intelligence influence learning success? A social cognitive neuroscience model. *Social Cognitive and Affective Neuroscience, 1*(2), 75–86.

Martin, A. J. (2006). The relationship between teachers' perceptions of student motivation and engagement and teachers' enjoyment of and confidence in teaching. *Asia- Pacific Journal of Teacher Education, 34*(1), 73–93.

Martino, A. M., & Maher, C. A. (1999). Teacher questioning to promote justification and generalization in mathematics: What research practice has taught us. *The Journal of Mathematical Behavior, 18*(1), 53–78.

Merritt, E. G., Palacios, N., Banse, H., Rimm-Kaufman, S. E., & Leis, M. (2016). Teaching practices in grade 5 mathematics classrooms with high-achieving English learner students. *The Journal of Educational Research*, 1–15.

Moschkovich, J. N. (1999). Supporting the participation of English language learners in mathematical discussions. *For the Learning of Mathematics, 19*(1), 11–19.

Moschkovich, J. N. (2007). Using two languages when learning mathematics. *Education Studies in Mathematics, 64,* 121–144.

Moschkovich, J. N. (2011). Supporting mathematical reasoning and sense making for English learners. In M. Strutchens & J. Quander (Eds.), *Focus in high school mathematics: Fostering reasoning and sense making for all students* (pp. 17–36). National Council of Teachers of Mathematics.

Moschkovich, J. N. (2013). Principles and guidelines for Equitable Mathematics teaching practices and materials for English language learners. *Journal of Urban Mathematics Education, 6*(1). https://doi.org/10.21423/jume-v6i1a204

Moser, J. S., Schroder, H. S., Heeter, C., Moran, T. P., & Lee, Y.-H. (2011). Mind your errors. *Psychological Science, 22*(12), 1484–1489. https://doi.org/10.1177/0956797611419520

Multi-tiered System of Supports (formerly RTI). (n.d.) American Institutes for Research. https://www.air.org/our-work/education/multi-tiered-system-supports-formerly-rti

Muncer, G., Higham, P. A., Gosling, C. J., Cortese, S., Wood-Downie, H., & Hadwin, J. A. (2022). A meta-analysis investigating the association between metacognition and math performance in adolescence. *Educational Psychology Review, 34*, 301–334.

National Council of Teachers of Mathematics. (2017). Positioning NCTM for a second century. https://www.nctm.org/secondcentury/

National Council of Teachers of Mathematics. (2014). Principles to actions: Ensuring mathematical success for all. https://www.nctm.org/PtA/Renkl, A., Atkinson, R. K., Maier, U. H., & Staley, R. (2002). From example study to problem solving: Smooth transitions help learning. *Journal of Experimental Education, 70,* 293–315.

Nelson, G., & Powell, S. R. (2018). A systematic review of longitudinal studies of mathematics difficulty. *Journal of Learning Disabilities, 51*(6), 523-539. https://doi.org/10.1177/0022219417714773

Nguyen, T., Watts, T. W., Duncan, G. J., Clements, D. H., Sarama, J. S., Wolfe, C., & Spitler, M. E. (2016). Which preschool mathematics competencies are most predictive of fifth grade achievement? *Early Childhood Research Quarterly, 36*, 550–560. https://doi.org/10.1016/j.ecresq.2016.02.003

Ohtani, K., & Hisasaka, T. (2018). Beyond intelligence: A meta-analytic review of the relationship among metacognition, intelligence, and academic performance. *Metacognition Learning, 13,* 179–212.

Pozas, M. & Schneider, C. (2019). Shedding light into the convoluted terrain of differentiated instruction (DI): Proposal of a taxonomy of differentiated instruction in the heterogeneous classroom. *Open Education Studies, 1,* 73–90.

Principles to actions: Ensuring mathematical success for all. (2014). National Council of Teachers of Mathematics.

Riccomini, P. J., & Morano, S. (2019). Guided practice for complex, multistep procedures in algebra: Scaffolding through worked solutions. *Teaching Exceptional Children, 51*(6), 445–454. https://doi-org.ezproxy.cul.columbia.edu/10.1177/0040059919848737

Rittle-Johnson, B., Fyfe, E. R., Hofer, K. G., & Farran, D. (2017). Early math trajectories: Low-income children's mathematics knowledge from ages 4 to 11. *Child Development, 88*(5), 1727-1742. https://doi.org/10.1111/cdev.12662

Root, J. R., Cox, S. K., Saunders, A., & Gilley, D. (2019). Applying the Universal Design for Learning framework to mathematics instruction for learners with extensive support needs. *Remedial and Special Education, 41*(4), 194–206. https://doi.org/10.1177/0741932519887235

Rose, D. H., & Meyer, A. (2002). *Teaching every student in the digital age: Universal Design for Learning.* Association for Supervision and Curriculum Development.

Roy, A., Guay, F., & Valois, P. (2013). Teaching to address diverse learning needs: Development and validation of a differentiated instruction scale. *International Journal of Inclusive Education, 17*(11), 1186–1204.

Russo, J., Bobis, J., Downton, A., Hughes, S., Livy, S., McCormick, M., & Sullivan, P. (2020). Elementary teachers' beliefs on the role of struggle in the mathematics classroom. *The Journal of Mathematical Behavior, 58,* 100774.

Ryan, R. M., & Deci, E. L. (2000). Intrinsic and extrinsic motivations: Classic definitions and new directions. C*ontemporary Educational Psychology, 25,* 54–67. http://dx.doi.org/10.1006/ceps.1999.1020

Ryan, R. M., & Deci, E. L. (2020). Intrinsic and extrinsic motivation from a self-determination theory perspective: Definitions, theory, practices, and future directions. *Contemporary Educational Psychology, 61,* Article 101860. https://doi.org/10.1016/j.cedpsych.2020.101860

Sacco, D. M., Spies, T., & Pfannenstiel, K. (2022). Scaffolding language for learning in mathematics for English learners receiving intensive intervention. *Learning Disabilities Research & Practice, 37*(3), 204–215.

Saxe, G. B., & Sussman, J. (2019). Mathematics learning in language inclusive classrooms: Supporting the achievement of English learners and their English proficient peers. *Educational Researcher, 48*(7), 452–465.

Schlechty, P. C. (2001). *Shaking up the schoolhouse.* Jossey-Bass.

Shifrer, D., Phillippo, K., Tilbrook, N., & Morton, K. (2023). The relationship between ninth graders' perceptions of teacher equity and their math identity: Differences by student race and school racial composition. *Sociology of Education, 96*(2), 129–148. https://doi.org/10.1177/00380407221149016

Shinn, M. R., & Walker, H. M. (2010). *Interventions for achievement and behavior problems in a three-tier model including RTI.* National Association of School Psychologists.

Skinner, E. A., Pitzer, J. R., & Steele, J. S. (2016). Can student engagement serve as a motivational resource for academic coping, persistence, and learning during late elementary and early middle school? *Developmental Psychology, 52*(12), 2099–2117. https://doi.org/10.1037/dev0000232

Smith, M. S., & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions.* National Council of Teachers of Mathematics.

Staples, M., & King, S. (2017). Eliciting, supporting and guiding the math: Three key functions of the teacher's role in facilitating meaningful mathematical discourse. In D. A. Spangler & J. J. Wanko (Eds.), *Enhancing classroom practice with research behind principles to actions* (pp. 25-36). Reston, VA: National Council of Teachers of Mathematics.

Steele, M. D., Raith, M. L., & Smith, M. S. (2017). *Taking action: Implementing effective mathematics teaching practices in grades 6–8.* The National Council of Teachers of Mathematics.

Stein, M. K., Smith, M.S., Henningsen, M. A., & Silver, E. A. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development.* Teachers College Press.

Sullivan, P., Knott, L., Yang, Y. (2015). The relationships between task design, anticipated pedagogies, and student learning. In A. Watson & M. Ohtani (Eds.), *Task design in mathematics education* (pp. 83–114).

Svinicki, M. D., & Vogler, J. S. (2012). Motivation and learning: Modern theories. In N. M. Seel (Ed.), *Encyclopedia of the sciences of learning* (pp. 2336–2339). Springer.

Sweller, J., & Cooper, G. A. (1985). The use of worked examples as a substitute for problem solving in learning algebra. *Cognition and Instruction, 2,* 59–89.

Sweller, J., van Merriënboer, J. J. G., & Paas, F. G. (1998). Cognitive architecture and instructional design. *Educational Psychology Review, 10,* 251–296.

Thijs, J., & Verkuyten, M. (2009). Students' anticipated situational engagement: The roles of teacher behavior, personal engagement, and gender. *The Journal of Genetic Psychology, 170*(3), 268–286.

The University of Texas at Austin Center for Teaching and Learning. (2024.). *Metacognition.* https://ctl.utexas.edu/metacognition

Tomlinson, C. A. (2014). *The differentiated classroom: Responding to the needs of all learners.* Association for Supervision and Curriculum Development.

Torres-Velasquez, D., & Lobo, G. (2005). Culturally responsive mathematics teaching and English language learners. *Teaching Children Mathematics, 11*(5), 249–255.

Trafton, J. G., & Reiser, B. J. (1993). The contributions of studying examples and solving problems to skill acquisition. In M. Polson (Ed.), *Proceedings of the 15th Annual Conference of the Cognitive Science Society* (pp. 1017–1022). Erlbaum.

U.S. Department of Education (2018). *A leak in the STEM pipeline: Taking algebra early.* www2.ed.gov/datastory/stem/algebra/index.htm l

Van de Walle, J., Karp, K., Lovin, L., & Bay-Williams, J. (2018). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades 3–5* (3rd ed.). Pearson.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes.* Harvard University Press.

Walker, A., & Leary, H. (2009). A problem based learning meta analysis: Differences across problem types, implementation types, disciplines, and assessment levels. *The Interdisciplinary Journal of Problem-based Learning, 3*(1), 12-43.

Wang, M., Binning, K. R., Del Toro, J., Qin, X., & Zepeda, C. D. (2021). Skill, thrill, and will: The role of metacognition, interest, and self-control in predicting student engagement in mathematics learning over time. *Child Development, 92*(4), 1369–1387.

Warshauer, H. K. (2015). Productive struggle in middle school mathematics classrooms. *Journal of Mathematics Teacher Education, 18*(4), 375-400. https://doi.org/10.1007/s10857-014-9286-3

Wiliam, D. (2017). *Embedded formative assessment.* Solution Tree Press.

Wirkala, C., & Kuhn, D. (2011). Problem-based learning in K–12 education: Is it effective and how does it achieve its effects? *American Educational Research Journal, 48*(5), 1157–1186.

Wong, Z. Y., Liem, G. A. D., Chan, M., & Datu, J. A. D. (2024). Student engagement and its association with academic achievement and subjective well-being: A systematic review and meta-analysis. *Journal of Educational Psychology, 116*(1), 48–75. https://doi.org/10.1037/edu0000833

Woolfolk, A., & Margetts, K. (2007). *Educational psychology.* Pearson, Prentice Hall.

Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). *Principles for the design of mathematics curricula: Promoting language and content development.* Retrieved from Stanford University, UL/SCALE website: http://ell.stanford.edu/content/mathematics-resources-additional-resources

3rd Annual International Seminar on Transformative Education and Educational Leadership (AISTEEL 2018), Medan City, Indonesia. https://doi.org/10.2991/aisteel-18.2018.101

Visit us online for more information on Amplify Desmos Math.

amplify.com/math

Amplify.

1649869